SEQUENCES OF WET OR DRY DAYS DESCRIBED BY A MARKOV CHAIN PROBABILITY MODEL

LEONARD L. WEISS

U.S. Weather Bureau, Washington, D.C.

ABSTRACT

A Markov chain probability model is shown to fit sequences of wet or dry days in records of various length and for several climatically different areas. Seasonal variation of the probability values is shown, but no apparent secular trend. A nomograph relating probability, length of sequence, and cumulative probability is presented.

1. INTRODUCTION

Besson [1] pointed out that in the 50 years of observations at Montsouris, France, the number of observed sequences of rainy days did not agree with that calculated on the basis of a constant probability equal to the ratio of the number of rainy days to the total number of days of observation. The observation showed too few short sequences and too many long ones. He drew the conclusion that the probability of a rainy day occurring was not independent of past conditions.

Weiss [13], in an investigation of the duration of stormy periods and the intervals between them for four 10° "squares" in the Northern Hemisphere (near England, Newfoundland, the Great Lakes, and the Aleutians), found fewer short sequences and more long ones than constant probability would indicate. This suggests that Besson's conclusions apply to the weather over an area as determined by an analyst from the synoptic weather map, as well as to that at a single station.

Jorgensen [9], in his study of persistency of rain and no-rain periods at San Francisco (20-yr. record of winter precipitation) similarly found fewer short sequences and more long ones than expected by chance when the (constant) probability of chance occurrence was defined as the ratio of rain days to total days of observation. He concluded that weather persistence was a real meteorological phenomenon and should be considered in making or verifying forecasts.

Williams [14], after remarking that previous investigators had demonstrated that sequences of wet or of dry days "have a certain statistical characteristic . . .; namely that the longer the spell has lasted the more likely it is to last another day," successfully applied a logarithmic series to fit the frequency distributions at Harpenden (Rothamsted Experimental Station), England, for the 10-yr. period 1938-47. By breaking the 10-yr. record into two 5-yr. periods, computed separately, he suggested that there was no secular trend in the frequency distribution.

Longley [10] concluded from his study of the length of wet and dry spells at Canadian cities that the probability of a wet day, given the previous day wet, is constant no matter how long the wet period has persisted, and similarly for the weather following a dry day except for a slight increase in the probability of dry weather with increasing length of the dry period. He defines the frequency (y) of wet or dry periods of n days or longer as

$$\log y = a + bn \tag{1}$$

fitted by least squares. The values a and b are constants for a given station and type (wet or dry) of series. The equation can then be used to determine probabilities. He presents the equation fitted to the count of dry periods and of wet periods for March at Montreal from 1874 to 1951.

He suggests an alternative method (which gives somewhat different results) for determining the probabilities, but either method when applied to monthly data for five Canadian cities demonstrated a seasonal variation in the probabilities. However, he points out that the probabilities do not change much with length of record and suggests that approximately correct values might be obtained even with less than 30 yr.

Cooke [5] found, in his examination of the 50-yr. (1900–1949) rainfall records at Moncton, New Brunswick, that while the wet spells could be fitted by a "Williams" logarithmic series, the dry spells *could not*, but were fitted satisfactorily by a simple geometric series. His tabulated data indicate a definite seasonal variation for each type (wet or dry) series.

2. THE MARKOV CHAIN PROBABILITY MODEL

In 1962, Gabriel and Neumann [7] in their study of sequences in daily rainfall occurrence at Tel Aviv (27 seasons) found them to be well described by a Markov chain probability model.

This model assumes that the probability of rain occurring on any day depends only on whether it did or did not occur on the previous day. The amount of rainfall is involved only in the definition of occurrence or non-occurrence. This probability model is referred to as a Markov chain whose parameters are the two conditional probabilities p_0 and $(1-p_1)$, where p_0 is the probability of a wet day, given the previous day dry, and $(1-p_1)$ is the probability of a dry day, given the previous day wet:

$$p_1=Pr\{W|W\}; \qquad (1-p_1)=Pr\{D|W\} \tag{2}$$

$$p_0=Pr\{W|D\}; \qquad (1-p_0)=Pr\{D|D\} \tag{3}$$

from which the probability of a dry spell of length n is

$$p_0(1-p_0)^{n-1} \tag{4}$$

and of a wet spell of length n

$$(1-p_1)p_1^{n-1} \tag{5}$$

The cumulative distribution through n is, for wet sequences

$$1-p_1^n \tag{6}$$

and for dry sequences

$$1-(1-p_0)^n \tag{7}$$

The probability for dry sequences greater than n is

$$(1-p_0)^n \tag{8}$$

and of wet sequences greater than n is

$$p_1^n \tag{9}$$

3. RESULTS OF APPLICATION TO OTHER DATA

The success of Gabriel and Neumann with the Markov chain model at Tel Aviv prompted me to apply it to the data presented by the aforementioned investigators. Comparisons of the sequences computed using the Markov chain probability model and those reported in the original papers are presented in tables 1 and 2. The data of Besson (table 1) are plotted in figure 1 for a sample visual comparison.

It seems apparent that this probability model is rather successful in giving a very close representation of the frequency of sequences of wet or dry days reported by several investigators in localities having very different climates.

Caskey [2, 3, 4] applied the Markov chain probability model to Topil's [12] data for Denver, Colo., to Miller's [11] data for Des Moines, Iowa, and to Hilsmeier's [8] data for Oak Ridge, Tenn., to compute sequence frequencies which were found to agree very well with those observed.

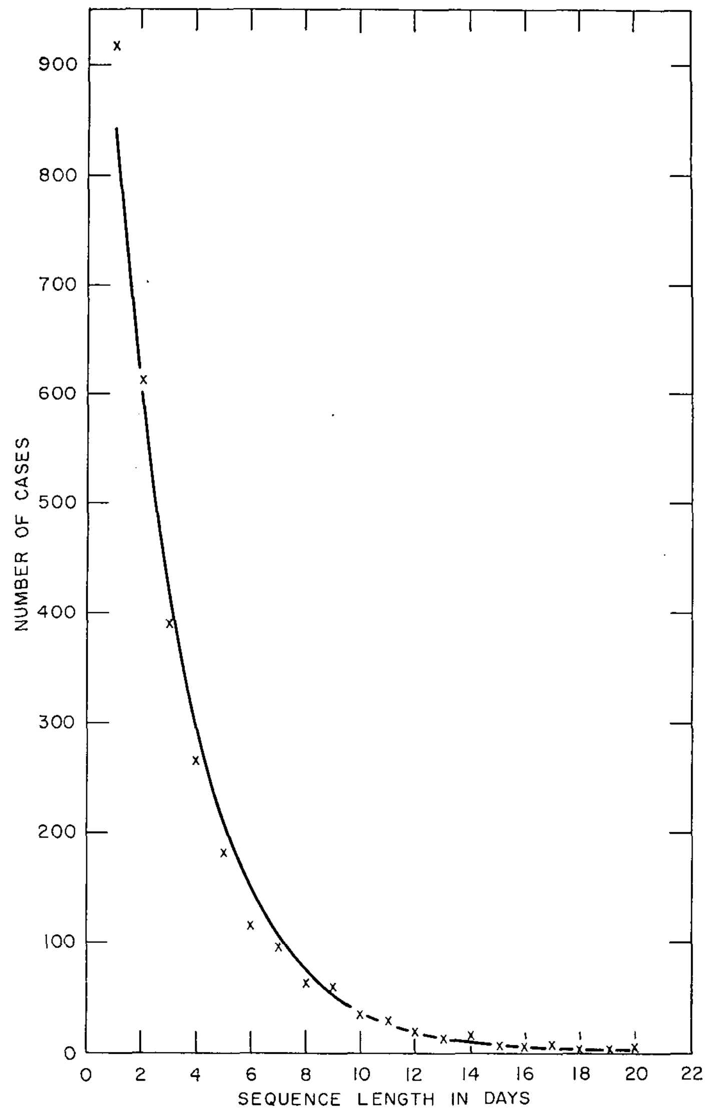

FIGURE 1.—Reported sequence length frequency versus computed sequence length frequency at Montsouris (Besson's [1] data). The observed frequency is shown by x's, that computed by the probability model shown by the solid line.

The magnitude of the rainfall amount used to dichotomize the record does not enter the model directly. It is however, reflected in the probability parameters. That is to say, the probability of a wet day, given the previous day dry, will be much smaller for a wet day defined by the occurrence of an inch or more of precipitation than for one defined by the occurrence of one-tenth of an inch. This is illustrated by the application of this model to the counts of sequences of dry days in four categories (precipitation <0.01, <0.10, <0.50, and <1.00 in.) at Kansas City for the 50-yr. period 1912–61; it shows satisfactory fit with all categories. Table 3 gives the computed and observed values.

TABLE 1.—*Comparison of observed sequences (O) of wet (W) and/or dry (D) days with those computed (C) by a Markov chain probability model*

Investigator	Station	Period of record		Run Length (days) 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	Total
Besson [1]	Montsouris	1873	WC	842	592	418	296	207	145	102	72	50	36	25	18	13	9	6	4	3	2	2	1	1	*	*	*	*	*	*	*	*	*	*	
$p_1=0.704$		to	WO	917	614	389	263	181	117	99	63	59	34	27	19	14	14	6	6	6	1	2	4	3	0	0	0	2	0	2	0	1	0	1	2844
		1922																																	
Jorgensen [9]	San Francisco.	1927	DC	60	48	39	31	26	21	17	14	11	9	7	6	5	4	3	2	2	2	1	1	1	1	1	*	*	*	*	*	*	*		
$p_0=0.192$		to	DO	86	51	37	27	13	11	14	7	7	12	11	7	5	0	4	3	2	1	1	2	2	1	1	0	1	0	1	2	1	1		311
$p_1=0.606$		1947	WC	121	73	43	27	16	10	6	4	2	1	1	*	*	*	*	*	*	*	*													
			WO	128	70	38	20	21	12	5	7	0	2	0	1	0	1	1	0	0	0	1													307
Williams [14]	Harpenden	1938	DC	214	141	93	62	41	27	16	12	8	5																						
$p_0=0.339$		to	DO	281	137	66	26	35	24	13	8	7	8																						630
$p_1=0.648$		1947	WC	233	144	94	61	39	25	16	11	7	4																						
			WO	253	146	87	38	34	23	17	6	4	4																						632
Longley [10]	Montreal	1874	DC	184	116	73	46	29	18	11	9	4	3	2	1	1	*	*	*	*	*														
$p_0=0.37$		to	DO	192	117	75	41	26	17	6	6	5	6	0	2	0	1	0	0	1	1														496
$p_1=0.56$		1951	WC	216	121	68	38	21	12	7	4	2	1	1	*	*	*	*																	
			WO	196	139	81	34	20	8	5	5	3	0	0	0	0	0	1																	492
Cooke [5]	Moncton	1900	DC	981	710	515	373	270	195	141	102	74	54	39	28	20	15	11	8	6	4	3	2												
$p_0=0.276$		to	WO	883	743	550	435	280	178	130	88	80	54	38	25	17	13	9	7	9	3	2	2												3552
$p_1=0.338$		1949	WC	2350	796	271	92	31	10	4	1	*	*																						
			WO	2425	758	203	92	38	18	5	6	3	3																						3552

*less than 0.5

Application of the model to a 50-yr. record (1912–61) at Fort Worth provides the values shown in table 4 for the <0.01 in. category. Again the Markov chain model fits the observed distribution very well.

TEST FOR SECULAR TREND

Williams data indicated no secular trend in the probabilities in the 10 yr. of data he used. Longley suggested something less than 30 yr. as an adequate length of record to give stable values of the probabilities. The Kansas City and Fort Worth data (50 yr.) were divided into two 25-yr. periods to test for secular variation. Table 5 shows the p_0 probability values (precipitation <0.01 in.) determined from the two periods separately. This indicates there is relative secular stability of the probabilities.

SEASONAL VARIATION

The data of table 5 suggest a seasonal variation in the probabilities. This was also noted in Cooke's data and in Longley's data. Table 6 gives the p_0 and $1-p_1$ probabilities for the five Canadian stations studied by Longley. The magnitude of this seasonal variation suggests it must be taken into account.

In addition, attention is called to the seasonal variation in the p_0 and $1-p_1$ values for East Lansing, Mich., based on 91 yr. of record reported by Eichmeier and Baten [6] from which table 7 has been taken.

TABLE 2.—*Comparison of sequences of duration of stormy periods and intervals between periods observed for four areas and those computed by Markov chain probability model. Observed data for Area I (10° "square" centered at 55° N., 5° W.), Area II (centered at 45° N., 55° W.), Area III (centered 45° N., 85° W.) and Area IV (centered 55° N., 165° W.), 1933–1938, reported by Weiss [13].*

Run Length (days)	AREA I				AREA II				AREA III				AREA IV			
	$p_1=0.592$		$p_0=0.453$		$p_1=0.584$		$p_0=0.512$		$p_1=0.668$		$p_0=0.490$		$p_1=0.488$		$p_0=0.400$	
	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.
1	140	138	162	178	135	124	188	207	99	73	170	192	178	170	144	137
2	83	92	89	85	79	82	92	97	66	72	87	97	87	90	86	98
3	49	37	48	45	46	42	45	38	44	41	44	32	42	46	52	51
4	29	27	27	17	27	34	22	18	30	33	22	9	21	22	31	40
5	17	22	14	12	16	9	11	2	20	26	11	12	10	6	19	11
6	10	5	8	6	9	9	5	2	13	17	6	2	5	9	11	7
7	6	8	4	7	5	8	3	0	9	10	3	1	2	2	7	5
8	4	5	2	3	3	2	1	2	6	7	2	0	1	2	4	3
9	2	4	1	3	2	6	1	0	4	5	1	0	1	0	2	2
10	1	1	1	1	1	3		1	3	4	*	0	*	0	1	4
11	1	1			1	2			2	3	*	1	*	1	1	1
12	1	1			*	1			1	3					1	0
13	*	0			*	1			1	2					*	0
14	*	1			*	0			1	0					*	0
15	*	1			*	0			*	1					*	0
16					*	1			*	0						
17					*	0			*	1						
18					*	1										
Total		343		357		325		367		298		346		348		359

*Less than 0.5.

TABLE 3.—*Comparison of sequences of dry days (in four categories) observed at Kansas City, 1912–1961 and those computed by Markov chain probability model*

A. Precipitation <0.01 in.

Run Length (days)	January $p_0=0.192$ Comp.	Obs.	April $p_0=0.300$ Comp.	Obs.	July $p_0=0.221$ Comp.	Obs.	October $p_0=0.185$ Comp.	Obs.
1	41	45	79	74	55	51	39	40
2	33	33	55	62	43	42	32	31
3	27	31	39	30	34	30	26	27
4	22	26	27	35	26	32	21	20
5	18	13	19	14	20	28	17	13
6	14	9	13	16	16	10	14	17
7	11	7	9	11	13	13	11	10
8	9	8	6	6	10	11	9	9
9	8	3	5	6	8	8	8	4
10	6	5	3	2	6	4	6	8
11	5	4	2	4	5	5	5	3
12	4	6	2	1	4	3	4	5
13	3	4	1	1	3	1	3	2
14	3	1	1	1	2	2	3	3
15	2	6	1	1	2	4	2	4
16	2	4	*	0	1	2	2	1
17	1	1			1	1	2	2
18	1	3			1	1	1	2
19	1	3			1	0	1	1
20	1	0			*	0	1	1
21	1	0			*	1	1	0
22	*	0			*	1	1	0
23	*	0			*	0	*	1
24	*	0			*	0	*	2
25	*	0			*	0	*	0
26	*	0			*	1	*	1
27	*	0					*	1
28	*	1					*	0
29	*	1					*	0
30	*						*	0
31	*						*	1
Total		214		264		251		209

B. Precipitation <0.10 in.

Run Length (days)	January $p_0=0.0957$ Comp.	Obs.	April $p_0=0.194$ Comp.	Obs.	July $p_0=0.148$ Comp.	Obs.	October $p_0=0.129$ Comp.	Obs.
1	13	14	44	37	28	25	21	25
2	12	15	36	41	24	17	18	15
3	11	12	29	33	20	22	16	9
4	10	14	23	33	17	12	14	18
5	9	10	19	14	15	21	12	10
6	8	5	15	15	13	11	11	15
7	7	5	12	10	11	13	9	10
8	6	5	10	9	9	9	8	9
9	6	5	8	5	8	10	7	8
10	5	3	6	6	7	8	6	6
11	5	3	5	6	6	4	5	3
12	4	2	4	4	5	8	5	5
13	4	3	3	2	4	3	4	4
14	3	3	3	4	3	4	3	3
15	3	5	2	4	3	3	3	3
16	3	3	2	1	2	2	3	1
17	3	2	1	0	2	4	2	2
18	2	3	1	2	2	4	2	1
19	2	1	1	0	2	0	2	3
20	2	1	1	0	1	0	2	1
21	2	3	1	1	1	1	1	0
22	2	0	*	0	1	2	1	1
23	1	1	*	1	1	0	1	1
24	1	3	*	0	1	1	1	1
25	1	0	*	0	1	1	1	0
26	1	0	*	0	*	1	1	0
27	1	0	*	1	*	0	1	1
28	1	1			*	1	1	1
29	1	2					*	0
30	1	0					*	1
31	1	0						
Over 31		9						7
Total		133		229		187		164

C. Precipitation <0.50 in.

Run Length (days)	January $p_0=0.0191$ Comp.	Obs.	April $p_0=0.0687$ Comp.	Obs.	July $p_0=0.0581$ Comp.	Obs.	October $p_0=0.0566$ Comp.	Obs.
1	1	0	7	4	5	2	5	8
2	1	1	6	5	5	5	4	3
3	1	3	6	3	5	11	4	2
4	1	2	5	6	4	7	4	4
5	1	1	5	3	4	6	4	6
6	1	1	5	4	4	4	3	6
7	1	0	4	5	4	6	3	2
8	1	0	4	8	3	4	3	3
9	1	0	4	6	3	4	3	1
10	1	0	3	1	3	1	3	2
11	1	0	3	5	3	2	3	1
12	1	0	3	2	3	4	2	2
13	1	0	3	1	2	6	2	2
14	1	2	3	1	2	1	2	5
15	1	0	2	4	2	1	2	1
16	1	0	2	2	2	3	2	0
17	*	1	2	3	2	1	2	1
18	*	0	2	2	2	2	2	0
19	*	2	2	4	2	0	2	3
20	*	1	2	2	2	2	2	0
21	*	0	2	2	2	2	1	0
22	*	2	1	1	1	1	1	1
23	*	0	1	2	1	0	1	3
24	*	0	1	5	1	0	1	1
25	*	0	1	3	1	1	1	0
Over 25		19		11		13		24
Total		35		95		89		81

D. Precipitation <1.00 in.

Run Length (days)	January $p_0=0.00454$ Comp.	Obs.	April $p_0=0.0233$ Comp.	Obs.	July $p_0=0.0239$ Comp.	Obs.	October $p_0=0.0252$ Comp.	Obs.
1	*	0	1	0	1	0	1	2
2	*	0	1	1	1	0	1	2
3	*	0	1	1	1	0	1	1
4	*	0	1	0	1	2	1	1
5	*	1	1	1	1	2	1	1
6	*	0	1	1	1	1	1	1
7	*	0	1	0	1	1	1	0
8	*	0	1	1	1	1	1	0
9			1	0	1	1	1	1
10			1	1	1	1	1	4
11			1	0	1	4	1	1
12			1	0	1	2	1	0
13			1	1	1	2	1	0
14			1	0	1	0	1	2
15			1	0	1	0	1	1
16			*	0	1	0	1	0
17			*	1	1	0	1	2
18			*	1	1	3	1	0
19			*	0	1	0	1	0
20			*	0	1	0	1	0
21			*	1	1	0	1	0
22			*	0	1	0	1	0
23			*	0	1	0	1	0
24			*	0	1	0	1	0
25			*	2	*	0	1	0
Over 25		8		20		17		19
Total		9		32		37		38

*less than 0.5

4. A NOMOGRAPH FOR CUMULATIVE PROBABILITY

The cumulative probability P of a sequence of n days either wet or dry may be obtained according to the Markov chain model, by use of formula (8) or (9) respectively. Conversely, the information wanted may be the length of sequence n that can be expected with some specified probability P. The following four formulas may be remembered for computing the length of dry sequence (n) for cumulative probabilities of 98, 90, 50, and 10 percent.

TABLE 4.—*Comparison of sequences of dry (<0.1 in.) days observed at Fort Worth, 1912–1961, and those computed by Markov chain probability model*

Run Length (days)	January $p_0=0.159$		February $p_0=0.178$		March $p_0=0.180$		April $p_0=0.206$		May $p_0=0.212$		June $p_0=0.146$	
	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.
1	36	43	33	34	39	33	46	54	51	59	27	27
2	30	28	27	32	32	35	36	27	40	43	23	21
3	25	21	22	25	26	27	29	31	32	20	19	18
4	21	26	18	21	21	19	23	27	25	22	17	16
5	18	20	15	8	17	30	18	17	19	21	14	12
6	15	16	12	11	14	13	14	10	15	13	12	9
7	13	6	10	4	12	6	11	14	12	7	10	15
8	11	9	8	10	10	8	9	10	10	12	9	7
9	9	9	7	6	8	9	7	11	8	7	8	1
10	7	7	6	3	6	6	6	5	6	5	6	8
11	6	8	5	3	5	6	5	4	5	7	5	6
12	5	2	4	4	4	1	4	1	4	6	5	5
13	4	2	3	5	4	3	3	1	3	2	4	2
14	4	4	3	5	3	0	2	3	2	3	3	4
15	3	5	2	3	2	3	2	0	2	1	3	5
16	3	6	2	0	2	2	1	2	1	1	2	1
17	2	2	1	2	2	2	1	0	1	0	2	2
18	2	1	1	1	1	0	1	0	1	2	2	3
19	2	1	1	0	1	1	1	3	1	3	2	2
20	1	3	1	0	1	1	1	0	1	1	1	1
21	1	0	1	2	1	0	*	0	*	1	1	2
22	1	2	1	0	1	0	*	0	*	1	1	1
23	1	0	*	1	*	0	*	1	*	0	1	1
24	1	1	*	0	*	2	*	0	*	0	1	0
25	1	0	*	0	*	0	*	0	*	0	1	0
Over 25		1		4		1				1		4
Total		223		184		208		221		238		173

Run Length (days)	July $p_0=0.118$		August $p_0=0.122$		September $p_0=0.120$		October $p_0=0.125$		November $p_0=0.129$		December $p_0=0.140$	
	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.
1	18	26	18	18	18	22	18	25	21	29	22	22
2	16	14	16	22	16	15	16	12	18	21	19	18
3	14	9	14	19	14	15	14	14	16	16	16	18
4	12	5	13	12	13	11	12	15	14	12	14	12
5	11	7	11	9	11	11	11	15	12	6	12	12
6	10	9	10	15	10	4	9	5	10	5	10	7
7	8	7	8	6	9	8	8	7	9	4	9	10
8	7	10	7	3	8	5	7	5	8	7	8	9
9	6	7	6	10	7	7	6	7	7	11	7	6
10	6	4	6	5	6	5	6	3	6	7	6	6
11	5	4	5	3	5	9	5	6	5	9	5	2
12	5	3	4	4	5	3	4	2	4	6	4	3
13	4	3	4	3	4	6	4	5	4	4	4	2
14	3	3	3	4	4	4	3	2	3	3	3	3
15	3	5	3	1	3	4	3	2	3	4	3	8
16	3	2	3	2	3	2	2	4	3	2	2	7
17	2	3	2	1	2	4	2	2	2	1	2	2
18	2	4	2	3	2	1	2	2	2	1	2	0
19	2	2	2	0	2	3	2	2	2	2	1	3
20	2	3	2	1	2	3	1	3	1	2	1	3
21	1	4	1	2	1	2	1	1	1	2	1	3
22	1	1	1	2	1	0	1	0	1	0	1	0
23	1	1	1	1	1	1	1	0	1	0	1	0
24	1	2	1	0	1	1	1	0	1	2	1	0
25	1	0	1	2	1	0	1	3	1	1	1	0
Over 25		13		3		6		5		4		3
Total		151		151		152		147		161		159

*Less than 0.5

TABLE 5.—*Probability values p_0 (precipitation <0.01 in.) for two 25-yr. periods.*

Period	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
						KANSAS CITY						
1912–36	0.204	0.181	0.223	0.294	0.322	0.280	0.218	0.236	0.254	0.211	0.161	0.166
1937–61	.181	.202	.244	.306	.326	.298	.223	.206	.186	.161	.153	.164
						FORT WORTH						
1912–36	.157	.176	.170	.194	.207	.142	.108	.130	.122	.129	.140	.140
1937–61	.161	.179	.191	.217	.216	.150	.129	.113	.118	.121	.118	.139

TABLE 6.—*Monthly probability values p_0 and $(1-p_1)$ for Canadian cities.* (*Extracted from Longley [10], table 4.*)

Station	Period of Record (yr.)		J	F	M	A	M	J	J	A	S	O	N	D
St. John, N.B.	69	p_0 $(1-p_1)$	0.43 .51	0.40 .51	0.39 .48	0.34 .49	0.33 .49	0.36 .47	0.35 .51	0.33 .47	0.33 .51	0.36 .52	0.38 .48	0.41 .50
Montreal, P.Q.	78	p_0 $(1-p_1)$	.50 .44	.43 .44	.37 .44	.32 .46	.33 .47	.35 .48	.35 .53	.33 .55	.33 .50	.34 .50	.43 .43	.46 .43
Winnipeg, Man.	76	p_0 $(1-p_1)$	.24 .61	.23 .59	.20 .62	.21 .56	.26 .56	.35 .54	.31 .62	.29 .59	.24 .57	.20 .58	.23 .56	.23 .58
Dawson, N.W.T.	50	p_0 $(1-p_1)$	.23 .57	.17 .62	.14 .66	.14 .69	.22 .54	.27 .49	.28 .48	.28 .50	.25 .55	.23 .56	.23 .50	.20 .56
Victoria, B.C.	52	p_0 $(1-p_1)$	.37 .28	.34 .32	.35 .34	.25 .51	.19 .51	.16 .56	.08 .58	.15 .49	.20 .44	.34 .30	.44 .24	.41 .23

$$n_{P=98}=\frac{-1.69897}{\log_{10}(1-p_0)}$$

$$n_{P=90}=\frac{-1.0000}{\log_{10}(1-p_0)}$$

$$n_{P=50}=\frac{-0.30103}{\log_{10}(1-p_0)}$$

$$n_{P=10}=\frac{-0.04576}{\log_{10}(1-p_0)}$$

For wet sequences, $\log_{10}p_1$ is substituted for $\log_{10}(1-p_0)$.

For convenience in practical use the nomograph in figure 2 was developed. It is entered on the sloping line labeled with the probability value (either p_0 or $(1-p_1)$). This sloping line is followed to the sequence length desired and the cumulative probability of all sequences to and including that length is read at the left side. Or, the sequence length can be ascertained by reversing the two final steps.

5. PROBABILITIES EXPRESSED AS RETURN PERIODS

The probabilities may also be expressed in terms of an average recurrence interval or return period T, given in years, of sequences of length greater than n days. That is to say, T is the ratio of the number of years of record to the total number of sequences of more than n days in length. For dry sequences this is

$$T_d=\frac{1-p_1+p_0}{sp_0(1-p_1)(1-p_0)^n} \tag{10}$$

and for wet sequences it is

$$T_w=\frac{1-p_1+p_0}{sp_0(1-p_1)p_1^n} \tag{11}$$

where s is the number of days in the subinterval for which the sequences are counted. For example, those sequences starting in September would require s to be 30, etc., while those of the entire year would have s equal to 365.

Equation (10) applied to the data for sequences of days with precipitation <0.01 in. at Kansas City and Fort Worth gives the results shown in table 8. This shows, for example, that only once in 100 yr., on the average, does Kansas City experience a sequence of more than 16 days in May for which the daily precipitation does not reach 0.01 in.

TABLE 7.—*Monthly probability values p_0 and $(1-p_1)$ for East Lansing, Mich., (from Eichmeir and Baten [6], table 3)*

Station	Period of Record		April	May	June	July	Aug.	Sept.
East Lansing, Mich.	91 yr.	p_0 $1-p_1$	0.288 .518	0.294 .521	0.277 .573	0.254 .631	0.242 .652	0.257 .590

TABLE 8.—*Dry sequence lengths n (in days) corresponding to given return periods T_d for <0.01-in. category*

T_d (years)	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
	KANSAS CITY											
100	29	28	23	18	16	18	25	25	25	30	35	33
50	26	25	21	16	14	16	22	22	22	26	31	30
10	18	17	15	11	10	12	16	16	16	18	20	20
2	10	10	9	7	6	7	9	9	9	11	12	12
	FORT WORTH											
100	34	30	31	26	26	37	46	44	44	43	41	39
50	30	26	27	23	23	32	40	39	39	38	36	34
10	21	18	19	16	16	22	27	27	26	26	25	24
2	12	10	11	9	9	12	14	14	14	14	13	13

6. SUMMARY

The Markov chain probability model appears to apply equally well to sequence of rain days at Montsouris (50 yr.); to data on durations of and intervals between stormy periods in 10° square areas (3 yr.); to sequences of wet and dry days at San Francisco (20 yr.), Harpenden (10 yr.), Moncton (50 yr.), and Montreal (March only, 75 yr.); and to sequences of dry days at Kansas City (50 yr.) and Fort Worth (50 yr.).

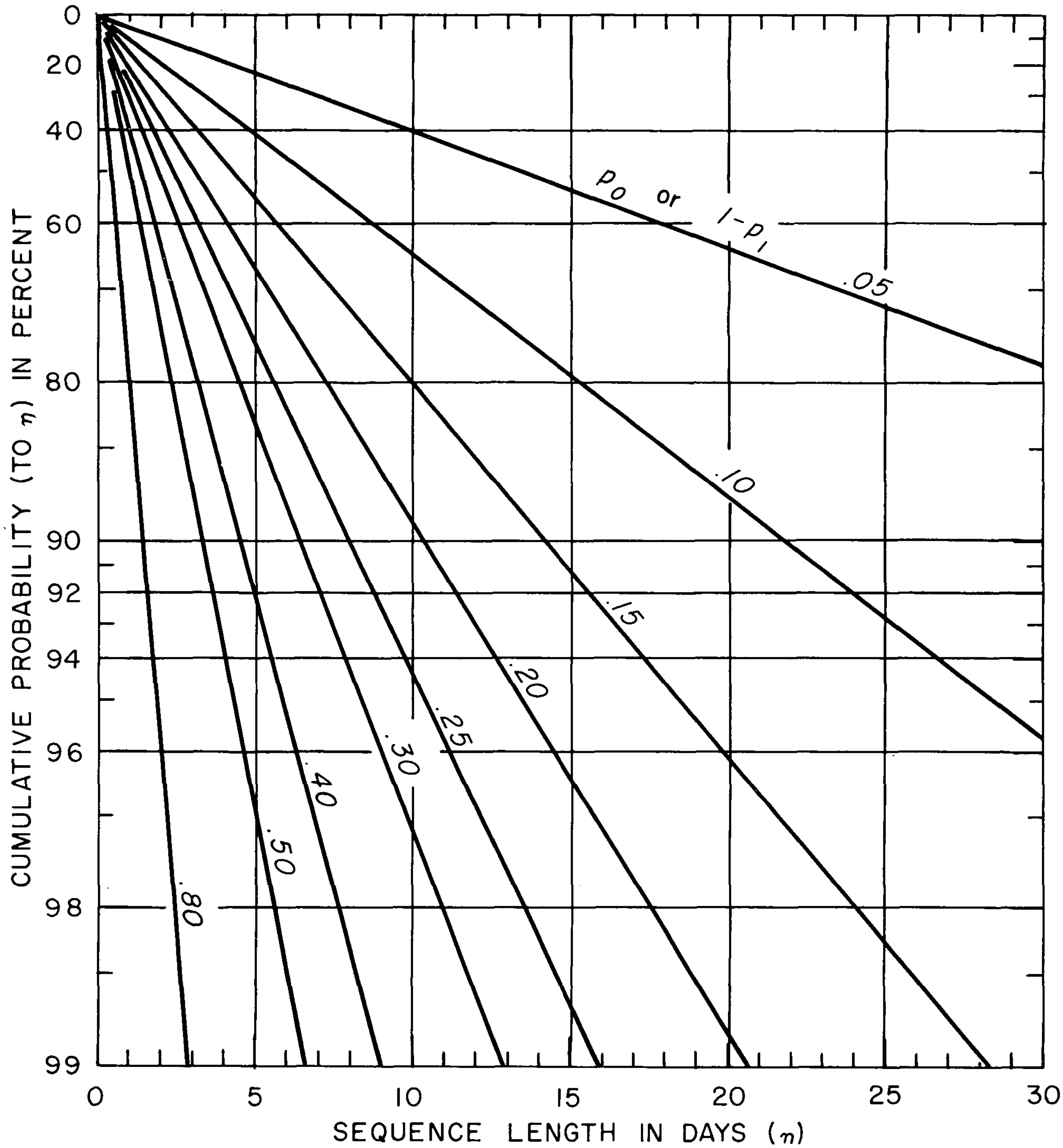

FIGURE 2.—Nomograph relating probability, length of sequence, and cumulative probability for dry or wet sequences. (Enter with p_0 or $1-p_1$ at right edge, then follow sloping line to sequence length desired, and read the cumulative probability of a sequence of that length at the left edge. Or sequence length can be ascertained by reversing the two final steps.)

The probabilities (p_0) computed at Kansas City and Fort Worth for the 25-yr. period 1912–36 showed little orderly or consistent change in the next 25-yr. period. There seems to be no definite appreciable secular trend, at these stations. However, the data do show a definite seasonal trend. The Canadian data of Longley [10] and Cooke [5] also indicated a seasonal trend.

A convenient nomograph was presented relating probability, length of sequence, and cumulative probability distribution, for dry or wet sequences.

It seems likely that the Markov chain model might be used to indicate the rainfall or drought probability regime of a station and from the results from many stations to specify it over a wide area (as on a map, say).

ACKNOWLEDGMENTS

I wish to thank my colleagues in the Hydrologic Services Division for reading the final manuscript and kindly proffering their editing suggestions.

REFERENCES

1. L. Besson, "Sur la probabilité de la Pluie," *Comptes Rendus*, t. 178, (19 Mai) 1924, pp. 1743–1745.
2. J. E. Caskey, Jr., "Comparison of Des Moines Precipitation Probabilities with Markov Chain Model Probabilities," U.S. Weather Bureau, Sept. 26, 1962 (unpublished manuscript).
3. J. E. Caskey, Jr., "A Markov Chain Model for the Probability of Precipitation Occurrence in Intervals of Various Length," *Monthly Weather Review*, vol. 91, No. 6, June 1963, pp. 298–301.
4. J. E. Caskey, Jr., "A Note on Markov Chain Model of Frequency of Consecutive Dry Days at Oak Ridge," U.S. Weather Bureau, June 1963 (unpublished manuscript).
5. D. S. Cooke, "The Duration of Wet and Dry Spells at Moncton, New Brunswick," *Quarterly Journal of the Royal Meteorological Society*, vol. 79, No. 342, Oct. 1953, pp. 536–538.
6. A. H. Eichmeier and W. D. Baten, "Rainfall Probabilities During the Crop Season in Southern Lower Michigan," *Monthly Weather Review*, vol. 90, No. 7, July 1962, pp. 277–281.
7. K. R. Gabriel and J. Neumann, "A Markov Chain Model for Daily Rainfall Occurrence at Tel Aviv," *Quarterly Journal of the Royal Meteorological Society*, vol. 88, No. 375, Jan. 1962, pp. 90–95.
8. W. F. Hilsmeier, "Supplementary Meteorological Data for Oak Ridge," ORO-199, U.S. Atomic Energy Commission, Oak Ridge, Tenn., Mar. 15, 1963, 57 pp.
9. D. L. Jorgensen, "Persistency of Rain and No-Rain Periods During the Winter at San Francisco," *Monthly Weather Review*, vol. 77, No. 9, Nov. 1949, pp. 303–307.
10. R. W. Longley, "The Length of Dry and Wet Periods," *Quarterly Journal of the Royal Meteorological Society*, vol. 79, No. 342, Oct. 1953, pp. 520–527.
11. S. R. Miller, "Des Moines Precipitation Probability Study," U.S. Weather Bureau, Kansas City, Mo., June 1962 (unpublished manuscript).
12. A. G. Topil, "Precipitation Probability at Denver Related to Length of Period," *Monthly Weather Review*, vol. 91, No. 6, June 1963, pp. 293–297
13. L. L. Weiss, "Preliminary Report on Duration of Stormy Periods at Selected Localities and Intervals Between Periods," *Research Paper* No. 3, U.S. Weather Bureau, Washington, Jan. 1944.
14. C. B. Williams, "Sequences of Wet and Dry Days Considered in Relation to the Logarithmic Series," *Quarterly Journal of the Royal Meteorological Society*, vol. 78, No. 335, Jan. 1952, pp. 91–96.

[Received August 7, 1963; revised November 1, 1963]